Columns

Problem No. 1
What is the critical buckling load for a steel hinge-hinge column, 12' long, with a pipe cross-section of external diameter of 4" and thickness ¼"? Do not use safety factors.

$$I = \frac{\pi D_{out}^{\ 4}}{64} - \frac{\pi D_{in}^{\ 4}}{64} = \frac{3.14 \times 4^4}{64} - \frac{3.14 \times 3.5^4}{64} = 5.2 in^4$$

$$P_{crit} = \frac{\pi^2 EI}{L^2} = \frac{3.14^2 \times 29{,}600{,}000 \times 5.2}{(12 \times 12)^2} = 73{,}263 lb$$

Problem No. 2
What is the "slenderness" ratio of the column in problem No. 1?

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{I}{\frac{\pi}{4}\left(D_{out}^{\ 2} - D_{in}^{\ 2}\right)}} = \sqrt{\frac{I}{\frac{3.14}{4}\left(4^2 - 3.5^2\right)}} = \sqrt{\frac{5.2}{2.944}} = 1.33 in$$

$$\frac{L}{r} = \frac{144}{1.33} = 108$$

Problem No. 3
What is the crushing load for the column in problem No. 1?

$$P_{crushing} = A \times f_y = 2.944 \times 36{,}000 = 105{,}975 lb$$

Problem No. 4
What is the length of the column in problem No. 1 that the crushing load is the same as the critical load for buckling?

$$P_{crushing} = P_{crit} = 105{,}975 lb = \frac{\pi^2 EI}{L^2} = \frac{3.14^2 \times 29{,}600{,}000 \times 5.2}{L^2}$$

$$L = \sqrt{\frac{3.14^2 \times 29{,}600{,}000 \times 5.2}{105{,}975}} = 119.7 in$$

Problem No. 5
A long column, made of laminated timber (E=2,000,000 psi), has a critical load of 5,000 lb. What would be the critical load of a column with the same properties made of pine that has a modulus of elasticity of 1,300,000 psi?

$$P_{crit} = \frac{\pi^2 EI}{L^2}$$

$$P'_{crit} = \frac{\pi^2 E'I}{L^2} \qquad \frac{P_{crit}}{P'_{crit}} = \frac{E}{E'} \qquad P_{crit} = P'_{crit} \times \frac{E}{E'} = 5{,}000 \times \frac{1{,}300{,}000}{2{,}000{,}000} = 3{,}250 lb$$

Problem No. 6
A long column has a critical load of 9,000 lb. What would be its critical load if it were 3 times longer, with everything else being equal?

$$P_{crit} = \frac{\pi^2 EI}{L^2}$$

$$P'_{crit} = \frac{\pi^2 EI}{L'^2} \Rightarrow \frac{P_{crit}}{P'_{crit}} = \frac{L'^2}{L^2} \Rightarrow P_{crit} = P'_{crit} \times \left(\frac{L}{L'} \right)^2 = 9{,}000 \times \frac{1}{3^2} = 1{,}000 lb$$

Problem No. 7

What is the ultimate axial load of a 12”x12” reinforced concrete cross-section with 4 No. 9 bars reinforcement? The strength of steel is 60 ksi and the compressive strength of concrete 3.6 ksi. Use a reduction factor of 0.70 (tied column) to obtain the ultimate strength from the nominal strength.

$$P_u = 0.70 \times P_o = 0.70 \times \left(A_s \times f_y + \left(A_c - A_s \right) \times f'_c \right) = 0.70 \times \left(4 \times 60 + \left(144 - 4 \right) \times 3.6 \right) = 520.8 kips$$