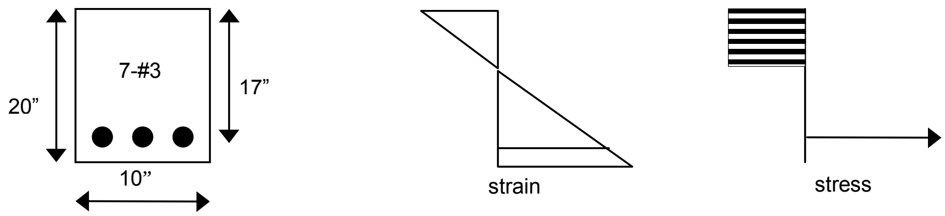
Moment

Problem 1

Determine the ultimate moment capacity of the section shown in the figure below, f'_c=3,000 psi and f_y=50,000 psi.



Given: b=10" f'_c=3000 psi

d= 17" f_y=50,000 psi

3 #7 ⇨ 3 steel bars, each with a diameter of $7\left(\frac{1}{8}\right)" = \frac{7}{8}"$

$$A_s = 3 \bullet \pi \left(\frac{D}{2}\right)^2 = 3\pi\left[\left(\frac{1}{2}\right)\left(\frac{7}{8}\right)\right]^2$$

$$\boxed{As = 1.804 \text{ in.}^2}$$

$$0.85\, f'_c\, \alpha\, b = C = T = A_s \,\square\, f_y$$

$$0.85\,(3000 \text{ psi})\, \alpha\, (10") = (1.804 \text{ in}^2)\,(50{,}000 \text{ psi})$$

$$\boxed{\alpha = 3.537 \text{ in.}}$$

$$M_{ULT} = 0.9\, M_{NOM} = 0.9\left(d - \frac{\alpha}{2}\right) T$$

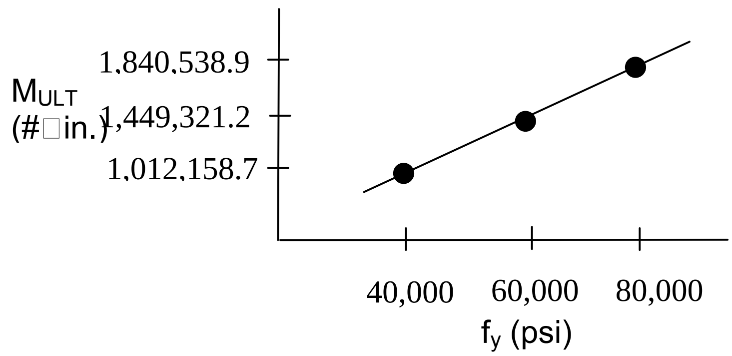
$$= 0.9\left(17" - \frac{3.537"}{2}\right)[(1.804 \text{ in.}^2)(50{,}000 \text{psi})]$$

$$\boxed{M_{ULT} = 1236493\ \#\square\text{in.}}$$

Problem 2

Repeat question no.1 for f_y = 40,000 psi, 60,000 psi and f_y = 80,000 psi. Plot the ultimate moment capacity of the strength of the steel.

f_y (psi)	α (in.)	M_{ULT} (#□in.)
40,000	2.8298	1,012,158.7
60,000	4.2447	1,449,321.2
80,000	5.6596	1,840,538.9



Problem 3

What is the strain of the steel and the strain on the top of the beam in the section shown in problem no.1, for the 3 grades of steel? Assume that the corresponding ultimate moment is applied in every case.

$$\varepsilon_s = \frac{0.003}{c}(d - c) = \frac{0.003}{c}(17\text{"} - c)$$

for $f'_c \leq 4$ ksi, $c = \frac{\alpha}{0.85}$ [use α values from No. 2]

f_y (psi)	c (in.)	ε_s	Check. (ductile failure) Is $\varepsilon_s > \varepsilon_y$?
40,000	3.3292	0.0123	yes
60,000	4.9938	0.0072	yes
80,000	6.6584	0.0047	yes

Problem 4

Determine the required area of steel of the cross-section shown in Fig. 1 if the factored bending moment is 1,000 kips-in. Use materials: f'_c=3,000 psi and f_y=40,000 psi. What bars should be used?

$$A_{s\,min} = \frac{200\text{ psi}}{f_y} \square b \square d = \frac{200}{40000}(10")(17")$$

$A_{s\,min}$ =0.85 in.

$0.85\, f'_c\, \alpha\, b = c = T = A_s f_y \Rightarrow \quad A_s = \frac{0.85\, f'_c\, \alpha\, b}{f_y}$

$$A_s = \frac{0.85\,(3000\text{ psi})\,\alpha\,(10^{11})}{40000\text{ psi}}$$

Assume an α value and check ultimate moment using:

$$M_{ULT} = 0.9\, M_{NOM} = 0.9\left(d - \frac{\alpha}{2}\right) \square T$$

$$10{,}000{,}000\ \#\square\text{in.} \geq 0.9\left(17" - \frac{\alpha}{2}\right) \square A_s\,(40{,}000\text{ psi})$$

α assummed	A_s (in.^{2})	M_{ULT} (#□in.)
1	0.6375	378,675
2	1.275	734,400
3	1.9125	1,067,175
2.75	1.7531	986,133

$A_s \approx 1.75$ in.^{2} ,

since M_{ULT} is close to the factored bending moment given. In the next lecture, you will learn how to find $A_{s\,max}$ to check if this answer falls in a proper range.